Impact of Consumer Reviews and Ratings on Sales, Prices, and Profits: Theory and Evidence

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Abstract

Many online retailers now offer consumer product ratings and reviews, and such information has become increasingly important to both retailers and consumers. However, extant empirical research shows mixed results regarding the effects of online ratings and reviews on product sales. We provide a theoretical framework to understand the impacts of consumer reviews and ratings on firms’ prices, sales, profits, and consumer surplus. We show how the economic impacts of consumer ratings may differ depending on the informativeness of the reviews, the quantifiability of the product attributes, and the competitive environment. Our analysis shows that the informativeness of reviews and the non-quantifiability of product attributes enhance the effects of product ratings. We find two interesting and counter-intuitive results. First, even though a monopolist will always benefit from an improvement in its product rating, a firm in a competitive market can *hurt* from an improvement in its product rating. When the low quality firm’s product rating improves, its equilibrium profit will *decrease* if its quality is above a threshold, and increase if its quality is below that threshold. Second, if the high quality firm’s product rating improves, *both* the high quality firm and the low quality firm will benefit. Our empirical findings based on point-and-shoot digital cameras and multivitamin data collected from Amazon.com provide strong support for our major results.

**Keywords:** Digital word-of-mouth, consumer reviews, e-commerce, online product reviews, vertical differentiation, pricing, competitive strategy, game theory
1. Introduction

Compared with offline retail channels, the Internet has limited capabilities in allowing consumers to try out products before making purchases. However, the digital channel offers such advantages as connecting consumers from around the world and allowing them to share their product knowledge and usage experiences. Ever since Amazon.com published its first consumer book review in 1995, an estimated 43% of online retailers now offer consumer reviews or ratings on their websites (Gogoi 2007), enabling consumers to inform one another on a variety of products from books, CDs, and electronics to apparel and jewelry. A recent study by eVoc Insights, a consumer experience consulting firm, reveals the significant impact of online reviews on consumer behaviors: 47% of shoppers read product reviews prior to their online purchases and 63% are more likely to buy from websites with online reviews or ratings (Gogoi 2007). Online consumer reviews have also penetrated offline channels playing an increasingly important role in influencing consumers’ offline purchase decisions of many products such as computers, electronics, automobiles, and movies. It is not surprising that online retailers and industry researchers alike have found that consumers trust online reviews more than other marketing communications such as newspapers, TV commercials, or promotional emails (Anonymous 2006, Campanell 2006).

Many researchers have studied both the positive and negative effects of traditional word of mouth (e.g., Banerjee 1992, Brown and Reingen 1987, Ellison and Fudenberg 1995, Katz and Lazarsfeld 2005, Mahajan et al. 1984, Reingen and Kernan 1986, Richins 1983). With the advent of the Internet and electronic commerce, online word of mouth has become of great interest to academic researchers. Dellarocas (2003 and 2005) provides an excellent discussion on the differences between online and traditional word of mouth, and gives an overview of the economic literature on reputation systems. Motivated by the popularity of online reviews, academic researchers have examined, both analytically and empirically, the effects of digital word of mouth on firms’ strategies and financial performance. Chen and Xie (2004) consider online consumer reviews as an element of a firm’s overall marketing communications mix and show that an online monopolistic seller can benefit or hurt from product reviews. Jiang and Chen (2007) show that with the availability of consumer reviews, both consumer surplus and
social welfare may be higher in a monopoly market than in a competitive market because a monopolistic vendor has less incentive than competitive vendors to induce (through lowered prices) consumers into buying an unmatched product. Product reviews also influence firms’ optimal advertising and pricing strategies; favorable third party reviews (e.g., from PC Magazine, Consumer Report, or CNET.com) may induce asymmetric advertising responses from the vendors depending on the market size of the taste-driven consumers and the review penetration rate (Chen and Xie 2005). Given the importance of online reviews, firms may have incentives to strategically manipulate their reviews. Dellarocas (2006) shows that competitive firms may be forced to spend resources on manipulating reviews because consumer perceptions will otherwise be biased against them. Mayzlin (2006) finds that a firm with inferior products will spend more resources on promotional chat activities than a firm with superior products.

Empirical evidence shows mixed results on the effects of consumer reviews and ratings. Dellarocas et al. (2004) find a high correlation between online and offline consumer ratings of movies and show that online user reviews within the first week of a new movie’s release can be used to provide good predictions for the movie’s total revenues. Chevalier and Mayzlin (2006) examine online book reviews and find that better reviews lead to higher sales. Clemons et al. (2006) use online reviews to assess firms’ differentiation strategies in the craft beer industry. Their research reveals that the variance in the ratings and the average of the top positive quartile of ratings are highly correlated with the sales growth, indicating the existence of hyper-differentiation and resonance marketing. However, contradictory empirical results also exist. For example, Duan et al. (2005) find that for movies, user ratings do not have any significant effect on their box office sales whereas the number of online postings influences box office sales. In contrast, Chen et al. (2006) find that for books, higher ratings are associated with higher sales and that highly informative reviews (i.e., those with a high proportion of helpful votes) strengthen this effect by creating additional sales. Overall, despite the widely believed importance of online reviews and ratings, academic research shows inconclusive results about their exact impact on firms’ strategies and resulting performances.
This paper provides a theoretical framework to understand the impacts of consumer reviews and ratings on firms’ prices, sales, profits, and consumer surplus. We construct an analytical model to illustrate the effects of product ratings, and show how such effects may differ depending on the characteristics of both reviews and product attributes as well as the competitive environment. To empirically test some of the key propositions from our analytical models, we collected consumer rating and price data on digital cameras and multivitamins sold on Amazon.com website. Results from our empirical analysis provide strong support for our propositions.

We address the following research questions: (1) Should firms charge higher prices for their highly rated products and how do the informativeness of reviews, the quantifiability of the product attributes, and the presence of competition affect their ability to exploit a rating increase? (2) How will firms’ sales volumes and profits change when one firm’s product rating changes? (3) How is the consumer surplus affected by product ratings? (4) Can the competitive firms be better off when their abilities to change prices (in response to rating changes) are limited?

This paper contributes to the current literature in several ways. First, it offers a simple theoretical framework to analyze the impacts of consumer reviews and ratings on firms’ prices, sales, and profits. It helps us understand why different empirical studies may show different results. Second, our research complements the extant theoretical research on online reviews and ratings. Our research focuses on the reviews and ratings of products rather than those of vendors (e.g., Brown and Morgan 2006, Livingston 2005, Pavlou and Dimoka 2006). We examine a vertically differentiated market rather than a horizontally differentiated market as considered by other researchers such as Chen and Xie (2004) and Jiang and Chen (2007). In addition, these researchers assume that consumers are uncertain about whether a product matches their preferences, and that the presence of online reviews will change, in specific and different ways depending on a consumer’s type and the informativeness of reviews, consumers’ ex ante beliefs about the probability of match and thereby affecting consumers’ product valuations. Our model does not make such assumptions on how consumers’ beliefs are updated differently. Finally, unlike much of the analytical literature (such as Chen and Xie 2004), this paper models the effects of both reviews and
ratings. To the best of our knowledge, this paper is the first in the extant literature to analytically examine how the impact of product ratings is affected by product characteristics in addition to the informativeness of reviews and the presence of competitors.

Our model builds upon the primitive that product reviews and ratings affect the perceived quality of a product. We decompose the perceived product quality into two parts arising from two types of product attributes—search versus experience/credence. The quality of the search attributes is easily communicated through product specifications and is perfectly known by consumers, whereas the quality of the experience/credence attributes is imperfectly known and is influenced by reviews and ratings. We find that when its product rating increases, a monopolistic firm will take advantage of the resulting increase in the perceived product quality by raising the price of its product and enjoying higher profits. Moreover, the firm will increase its price more if the reviews are more informative or if the product has a high proportion of experience/credence attributes. A higher product rating will never decrease a monopolist’s optimal sales volume.

In the competitive market, we examine two firms with quality differentiated products and find that an improvement in product ratings does not necessarily lead to increases in price, sales, or profits. Throughout the paper, we will refer to the firm with the higher overall perceived quality as the high quality firm and the other as the low quality firm. Interestingly, we find that an increase in the product rating of the high-quality firm will lead to higher prices, lower sales, and higher profits for both firms. Intuitively, the low quality firm benefits from its competitor’s increased rating because the increased rating of the high quality firm effectively makes it more differentiated from its competitor and hence price competition becomes less intense. In contrast, an increase in the low-quality firm’s rating reduces the difference between the perceived qualities of the two firms’ products and hence leads to more intensified competition, which leads to a price cut by the high quality firm and which definitely reduces the high quality firm’s equilibrium profit. However, the low quality firm’s optimal pricing strategy in response to the increase in its product rating depends on how differentiated the products are. For the low quality firm, an increase in its product rating has two opposing effects. On the one hand, the improvement in its rating
increases the perceived quality of its product leading to higher willingness-to-pay by consumers. On the other hand, its product now becomes less differentiated in the perceived quality space from its competitor’s product, and price competition between the two firms may hence be more intense. We find that there exists a threshold of the low quality firm’s perceived quality below which the low quality firm’s best strategy in response to an increase in its rating is to increase its price yielding higher equilibrium profits. If the low quality firm’s perceived quality is above this threshold, however, it will have to reduce its price because of the anticipated price cut by its competitor, and ultimately, the improvement in its product rating actually reduces its profits. We also find that though the consumer surplus always becomes larger when the low quality firm’s rating increases, an increase in the high quality firm’s rating may reduce the consumer surplus because the effect of lowered price competition may dominate the effect of consumers’ increased product valuation.

The potential detrimental effects to both firms when the low quality firm’s product rating improves are a direct result of both firms’ complete flexibility in adjusting their prices in response to changes to product ratings. We show that unless the high quality firm’s perceived quality is very high, both firms face the prisoner’s dilemma when the low quality firm’s rating increases. That is, each firm prefers to reduce its price to protect or increase its customer base, but in doing so, both firms are worse off. We find that in such situations, both firms are better off if neither firm has the ability to adjust its prices in response to rating changes than if both can freely adjust their prices.

The rest of our paper is organized as follows. In Section 2, we present our model and discuss the model assumptions. We analyze the impact of consumer product reviews and ratings in Section 3. Section 3.1 provides the analysis for a monopoly setting, and Section 3.2 shows the analysis in a competitive (duopoly) market. In Section 4, we present the empirical evidence for the testing of some of our major propositions. Section 5 concludes our paper and discusses possible future research.

2. Model

In our model, products are vertically differentiated (i.e., quality differentiated) rather than horizontally differentiated as in Chen and Xie (2005) or Jiang and Chen (2007). That is, all consumers
prefer a higher quality product if prices are the same. Our vertical differentiation model is similar to and builds upon the ones used by many researchers (e.g., Moorthy 1988, Mussa and Rosen 1978, Shaked and Sutton 1982) with the exception that in our model, the quality levels are formed from consumers’ perceptions based on product characteristics and are affected by consumer reviews and rating. The net consumer utility function is given by $u(v, P) = Q(r, \eta, \lambda) \cdot v - P$, where $P$ is the price of the product, $v$ represents the type of the consumer and $Q(r, \eta, \lambda)$ is a function of three parameters—the average rating ($r$) of the product, the informativeness ($\eta$) of the reviews, and the quantifiability ($\lambda$) of the product attributes. We assume that consumers are uniformly distributed in the type space; without loss of generality, we normalize the distribution to unity, i.e., $v \sim \text{uniform}[0,1]$. The product rating, denoted by $r$, is the average product rating from all reviews; the informativeness ($\eta$) of the reviews represents how informative or useful the reviews are. Researchers (Nelson 1970 and 1974; Darby and Karni 1973) are the first to classify goods and services into search, experience, and credence categories based on how readily consumers can determine the qualities of such goods and services. In this paper, we use a continuous parameter—quantifiability ($\lambda$)—to represent how quantifiable the product attributes are, and we operationalize $\lambda$ to be the proportion of relevant product attributes that are search attributes (and hence easily communicated through product specifications). Note that $Q(r, \eta, \lambda)$ represents the perceived quality of the product. In the real world, a true and objective quality level, even if it exists, is unlikely to be perfectly perceived ex ante by all consumers. Our model provides an alternative framework to model quality. Instead of assuming a perfectly observed, objective quality level, we assume that consumers form some perception about the product quality from product information including reviews and ratings. We make no explicit assumptions about to what value the perceived quality level converges. Researchers who believe the existence of a true, objective quality level may consider our perceived quality to eventually converge to that true quality level as consumers receive “perfect information” (product specifications, reviews and ratings, etc.). Practitioners may not be interested in whether a “true” quality level exists or what it is; they are most interested in consumers’ perceived quality level and how to
influence that level, because to them, consumers’ perception influences their willingness to pay and is hence a reality. Our model addresses practitioners’ needs by explicitly considering three concrete parameters that influence the perceived quality and by examining their impact on firms’ sales, prices, and profitability.

We decompose the perceived quality into two parts—one from the product’s search attributes and one from the experience/credence attributes: \[ Q(r, \eta, \lambda) = \lambda + (1 - \lambda)f(r, \eta). \] Consumers perfectly observe the search attributes based on product specifications, and infer the quality of the experience/credence attributes through consumer reviews and ratings. In assuming this functional form, we have normalized the average quality contribution from the search attributes to unity and thus, \( f(r, \eta) \), the quality contribution from the experience/credence attributes, is relative to that of the search attributes. We have also implicitly assumed that consumers have perfect knowledge about the search attributes even without review or rating information; this is very reasonable assumption because such attributes are usually clearly stated in the product specifications. Examples of search attributes are the number of pixels, the optimal zoom, and the LCD screen size of a digital camera, or the binding and the number of pages of a book. Examples of the experience/credence features are the image quality and ease of use of a digital camera or the writing quality of a book. Different products may differ in their quantifiability. For example, books contain mainly experience/credence attributes, blank CD-Rs or CD-RWs mainly search attributes, while digital cameras are somewhere in between. In essence, we have constructed a normalized, one-dimensional, perceived quality measure from the two types of relevant product attributes.

We make two assumptions on \( f(r, \eta) \), i.e., on how ratings and reviews affect consumers’ willingness to pay (WTP). First, \( f_r(r, \eta) > 0 \); that is, ceteris paribus, consumers’ WTP for a product increases as its rating increases.\(^1\) This is a reasonable assumption. A high product rating can be considered as a form of positive advertising; advertising, as assumed by many researchers (e.g., Adams

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\(^1\) In this paper, unless otherwise noted, a function with a letter subscript represents the partial derivative of the function taken with respect to that variable, e.g., \( f_r = \frac{\partial f}{\partial r} \) and \( f_{\eta r} = \frac{\partial^2 f}{\partial \eta \partial r} \).
and Yellen 1977), raises consumers’ reservation prices (i.e., WTP) for the product. Second, \( f_{\eta_r}(r, \eta) > 0 \); this implies that the positive effect of ratings on consumers’ WTP is stronger if the reviews are more informative. That is, other things being equal, consumers should be more willing to pay for a high-rating product if they find its reviews to be informative or trustworthy than otherwise. To empirically validate our assumptions, we conducted an informal study on 28 undergraduate students, asking them about their WTPs on digital cameras with different consumer ratings and informativeness of reviews. Results strongly support both of our assumptions. Note that we take reviews and ratings as exogenous in the sense that the micro dynamics of how reviews and ratings evolve is not modeled in this paper. That is, in our game, firms and consumers take reviews and ratings as given.

3. Analysis

3.1. Monopoly market

We first consider a monopolist firm offering a single product. We assume that the fixed cost of production is sunk. Let \( c \) denote the marginal cost of the product. A consumer of type \( v \) will buy the product if and only if \( u(v, P) \geq 0 \). Consumers have a utility of zero if they do not purchase the product. Each consumer buys at most one unit of the product. As discussed earlier, the perceived quality is given by \( Q(r, \eta, \lambda) = \lambda + (1 - \lambda)f_r(r, \eta) \), where \( \lambda \in (0,1) \), \( f_r(r, \eta) > 0 \) and \( f_{\eta_r}(r, \eta) > 0 \). The number of consumers who will purchase the product at price \( P \) is given by \( q(P) = 1 - \frac{P}{Q(r, \eta, \lambda)} \). The firm’s profit is

\[
\Pi(P) = (P - c)q = (P - c)\left(1 - \frac{P}{Q(r, \eta, \lambda)}\right).
\]

The maximum profit is achieved at \( P^* = \frac{c + Q(r, \eta, \lambda)}{2} \), which results in a sales volume (i.e., the total quantity of the product sold) of \( q^* = \frac{Q(r, \eta, \lambda) - c}{2Q(r, \eta, \lambda)} \) and

\[
\Pi^* = \frac{Q(r, \eta, \lambda) - c}{2Q(r, \eta, \lambda)}.
\]
**Proposition 1:** Suppose that the perceived quality of the monopolist’s product is given by
\[ Q(r, \eta, \lambda) = \lambda + (1 - \lambda) f(r, \eta), \]
where \( f_r(r, \eta) > 0 \) and \( f_{\eta r}(r, \eta) > 0 \). Then, a higher product rating leads to a higher optimal price and a higher profit. A higher product rating does not affect the optimal sales volume of information goods (for which \( c=0 \)), while it increases the sales volume for non-information goods.\(^2\)

**Proposition 2:** Suppose that the perceived quality of the monopolist’s product is given by
\[ Q(r, \eta, \lambda) = \lambda + (1 - \lambda) f(r, \eta), \]
where \( f_r(r, \eta) > 0 \) and \( f_{\eta r}(r, \eta) > 0 \). Then,

(a) The product rating’s effect on the firm’s optimal price is larger if its reviews are more informative; that is, \( \frac{\partial^2 P^*}{\partial \eta \partial r} > 0 \);

(b) The product rating’s effect on the firm’s optimal price is larger if the product has a smaller quantifiability factor; that is, \( \frac{\partial^2 P^*}{\partial \lambda \partial r} < 0 \).

Effectively, when a monopolist’s product rating increases, the firm can increase its price and make higher profits because the improved rating increases the perceived quality of the product and thereby consumers’ WTP. How much the firm’s price will increase in response to a rating improvement depends on how informative the reviews are and on the product’s quantifiability factor. The more informative are the reviews and the lower the product’s quantifiability factor, the higher the firm’s ability to exploit its product rating increase by charging a higher price. We next study a competitive market to analyze how one firm’s rating changes will affect its own sales, prices, and profits as well as those of its competitor. We will show that competitive firms may not necessarily benefit from improvements in their own product ratings and that they may actually benefit from a competitor’s rating improvement.

\(^2\) All proofs in this paper are provided in the Appendix.
3.2. Duopoly market

We now consider two firms each of which offers one product to compete in a vertically differentiated market. Their products have different perceived quality levels (e.g., due to differences in reviews, ratings, or some product attributes). A consumer of type $v$ derives a net utility of

$$u_i(v, P_i) = Q_i(r_i, \eta_i, \lambda_i) \cdot v - P_i$$

from product $i$ (of firm $i$), where $P_i$, $r_i$, $\eta_i$, and $\lambda_i$ are product $i$'s price, rating, review informativeness, and quantifiability, respectively. The perceived qualities of the two products are given by

$$Q_1(r_1, \eta_1, \lambda_1) = \lambda_1 + (1 - \lambda_1)f(r_1, \eta_1)$$

and

$$Q_2(r_2, \eta_2, \lambda_2) = \lambda_2 + (1 - \lambda_2)g(r_2, \eta_2),$$

respectively, where $f$ and $g$ satisfy the regularity assumptions: $f(r, \eta) > 0$, $g_{\eta r}(r, \eta) > 0$, and $g_{\eta r}(r, \eta) > 0$. Without loss of generality, we assume that product 1 has a higher perceived quality, i.e., $Q_1(r_1, \eta_1, \lambda_1) > Q_2(r_2, \eta_2, \lambda_2)$. Consumers derives zero net utility if they do not purchase any product.

Each consumer buys at most one product. A consumer of type $v$ will buy product $i$ if and only if

$$u_i(v, P_i) \geq 0 \quad \text{and} \quad u_i(v, P_i) > u_j(v, P_j) \quad \text{for} \quad j \neq i.$$  \textsuperscript{3} These two conditions are commonly known as the individual rationality and incentive compatibility constraint, respectively. Firms’ sales volumes are functions of their prices and the perceived qualities: $q_i(P_1, P_2; Q_1, Q_2).$ \textsuperscript{4} Firms’ profit functions are thus given by

$$\Pi_i = (P_i - c_i) \cdot q_i(P_1, P_2; Q_1, Q_2),$$

where $c_i$ denotes the marginal cost of firm $i$’s product. Firms maximize their profits by strategically (non-cooperatively) setting the prices of their products based on their perceived qualities. For ease of exposition, we assume that both products have zero marginal cost. It is straightforward to extend our analysis to non-zero marginal costs though the calculus becomes cumbersome; results from such an extension are qualitatively the same as long as the marginal costs are small compared with consumers’ maximum WTP.

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\textsuperscript{3} Without loss of generality, we assume that if consumers are indifferent between products, they will pick one with higher perceived quality.

\textsuperscript{4} To avoid notational clustering, we will not explicitly write out the arguments of functions $Q_i$ from now on.
The equilibrium prices, sales volumes and profits are given by Lemma 1. Propositions 3 and 4 summarize how the equilibrium will change when one firm’s product rating changes incrementally.⁵

**Lemma 1**: At equilibrium, firms’ prices are \( P_1^* = \frac{2(Q_1 - Q_2) \cdot Q_1}{4Q_1 - Q_2} \) and \( P_2^* = \frac{(Q_1 - Q_2) \cdot Q_2}{4Q_1 - Q_2} \); their sales volumes are \( q_1^* = \frac{2Q_1}{4Q_1 - Q_2} \) and \( q_2^* = \frac{Q_1}{4Q_1 - Q_2} \), yielding profits of \( \Pi_1^* = \frac{4(Q_1 - Q_2) \cdot Q_1^2}{(4Q_1 - Q_2)^2} \) and \( \Pi_2^* = \frac{(Q_1 - Q_2) \cdot Q_1 Q_2}{(4Q_1 - Q_2)^2} \), respectively.

**Proposition 3**: Effect of an incremental change in the high-quality product’s rating

If the high-quality firm’s product rating improves (deteriorates),

1. both firms’ equilibrium prices will increase (decrease);
2. both firms’ equilibrium sales volumes will decrease (increase);
3. both firms’ equilibrium profits will increase (decrease).

**Proposition 4**: Effect of an incremental change in the low-quality product’s rating

If the low-quality firm’s product rating improves (deteriorates),

1. its equilibrium price increases (decreases) if its perceived quality is relatively low
   (i.e., \( Q_2 < (4 - 2\sqrt{3})Q_1 \)) and decreases (increases) otherwise; its competitor’s price will always decrease (increase);
2. the equilibrium sales volumes of both firms will increase (decrease);

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⁵ We assume that the change in product ratings does not completely switch the relative quality perception of the products. For example, after an increase in the low-quality product’s rating, consumers will perceive the low quality product to have a higher quality than before, but its perceived quality is still assumed to be lower than that of the high-quality product.
Propositions 3 and 4 show what will happen if one firm’s product rating changes. We highlight two interesting and surprising results. First, if the firm with the higher perceived quality (i.e., firm 1) improves its product rating, both firms will benefit. If firm 1 keeps its price constant after an increase in its product rating, firm 2’s optimal response will be to lower its own price to prevent some of its customers from switching to firm 1’s product. Such anticipated price cut by firm 2 makes it suboptimal for firm 1 to keep its price fixed; in other words, stealing its competitor’s customers is not firm 1’s best strategy. Firm 1’s optimal strategy is to increase its price, which leads to slightly lower unit sales but a higher profit because of the increased profit margin. How large a price increase is optimal depends on the perceived qualities of the products as well as the size of the rating change. In response to firm 1’s price increase, firm 2 has some incentive to slightly increase its own price. It is somewhat counter intuitive that a low-quality firm would prefer an increase rather than a decrease in its competitor’s product rating. Essentially, an increase in the high-quality firm’s rating helps alleviate the pressure of price competition because it leads to further separation between the perceived qualities of the firms’ products.

Second, an improvement in the low-quality product’s rating can either benefit or hurt its firm’s own profitability. It is counter intuitive that a firm may actually be hurt by an increase in its product rating. Proposition 1 shows that in a monopoly setting, an increased product rating always improves the firm’s profit. In the competitive setting, however, the low-quality firm faces two opposing effects from its rating’s improvement. On the one hand, its improved rating means that consumers are more willing to pay for its product; this can potentially allow the firm to increase its price. On the other hand, its product is now closer, in the perceived quality space, to its competitor’s product; the reduced product differentiation increases the pressure for the firms to compete on price. Which of these two effects dominates depends on
the relative quality difference. If the low quality is low enough (i.e., $Q_2 < \frac{4Q_1}{7}$), the positive effect on consumers’ WTP plays a dominant role and the low quality firm’s profit will increase as a result of the improvement in its product rating. If $Q_2 > \frac{4Q_1}{7}$, however, the lower quality firm will suffer from an increase in its product rating; that is, intensified competition reduces both firms’ equilibrium profits even though their sales volumes both increase at equilibrium. In addition, though the high quality firm will always lower its price in response to an improvement in the low quality firm’s rating, the low quality firm’s optimal pricing strategy depends on the relative quality levels. If $Q_2 < (4 - 2\sqrt{3})Q_1$, the low quality firm will increase its price after its product rating improves; otherwise, its optimal price will be lower than before because of competitive responses.

Proposition 5 shows how changes in product ratings affect the consumer surplus. We find that consumers always benefit from a rating improvement of the low quality product because of increased valuation of the product and intensified price competition between firms. However, when the high quality product’s rating improves, consumers may be worse off because the effect of reduced price competition can dominate the effect of increased product valuation especially when the perceived product qualities are close.

**Proposition 5:** Effect of an incremental change in the product rating on consumer surplus

As the high-quality firm’s product rating improves (deteriorates), consumer surplus becomes larger (smaller) if $Q_2 < \frac{4Q_1}{5}$ and smaller (larger) if $Q_2 > \frac{4Q_1}{5}$. In contrast, as the low quality product’s rating improves (deteriorates), consumer surplus always becomes larger (smaller).

We have seen in Proposition 4 that when the low quality product’s rating improves, both firms may end up making less profits. This is because each firm will adjust its price responding to increased competition resulting from the closer product positions in the perceived quality space. What if firms cannot easily adjust their prices? Such inflexibility might potentially arise from menu costs or price
guarantees for previous customers. How will such pricing inflexibility affect both firms in the above situation? Proposition 6 answers this question. We find that unless firm 1’s perceived product quality is much higher than that of firm 2, both firms will be better off under inflexible pricing than under flexible pricing. Intuitively, this is because inflexible pricing restrains the otherwise intensive, direct price competition and hence allows firms to avoid the prisoner’s dilemma.

**Proposition 6:** Effect of pricing inflexibility when the low quality product’s rating improves

Suppose that the low quality product’s rating increases. The low quality firm will always be better off if both firms’ prices are fixed than if they can flexibly adjust. The high quality firm will see a smaller profit drop (as a result of the low quality firm’s rating improvement) under inflexible pricing than under flexibly pricing as long as its perceived quality is not too high (i.e., \(Q_1 < \frac{(2 + \sqrt{3})Q_2}{2}\)).

4. Empirical Evidence

4.1. Samples

To empirically validate some of our propositions, we collected, from Amazon.com website, product, consumer rating, and price data for point-and-shoot digital cameras from March 4 to 6, 2007 and for multivitamins from April 19 to 20, 2007, using an automated data collection agent designed by following Allen et al.’s (2006) recommendations.\(^6\) We selected these two product categories to represent products with different quantifiabilities. Since Amazon.com does not release data on its sales and relative sales ranks may remain unchanged even when the actual sales change, we can test only our propositions regarding prices. Because Amazon.com not only sells its own products but also allows third-party sellers to sell their products on its website, our data were pricing data from multiple sellers. We summarize the sample descriptive statistics in Table 1.

To determine whether a product is of high or low quality, we first divided each sample into groups of similar qualities based on their search attributes. For example, we divided digital cameras into 33 groups, each group with similar resolution in mega pixels and optical zoom. For multivitamins, we

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\(^6\) The different durations of data collection were due to sample size differences for the two product categories.
divided the data into 13 groups based on the market segmentation (adults, men, women, vegetarians, senior, children, etc.), whether the product is chewable or not, and whether it contains some additional ingredients such as irons and whole food. We identified a product as a high quality one if its average rating was higher than the sample average. The sample average consumer rating was 3.80 for digital cameras and 4.28 for multivitamins. As a result, digital cameras with average consumer ratings of 4 stars or higher were considered high quality products (within their groups), while multivitamins with average consumer ratings of 4.5 stars or higher were high quality ones. Within each group, we examined how the average consumer rating of a product, as well as the average rating of products with the opposite quality in that group, affects the price of the product.

Table 1. Sample Descriptive Statistics.

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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mega Pixels</td>
<td>5.49</td>
<td>1.71</td>
<td>1.00</td>
<td>10.10</td>
</tr>
<tr>
<td>Optical Zoom</td>
<td>3.11</td>
<td>1.84</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>Average Star Rating</td>
<td>3.80</td>
<td>0.91</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Price</td>
<td>234.85</td>
<td>141.06</td>
<td>22.95</td>
<td>899.80</td>
</tr>
<tr>
<td><strong>Multivitamins (N₂=139)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Star Rating</td>
<td>4.28</td>
<td>1.15</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Price</td>
<td>20.49</td>
<td>11.13</td>
<td>5.35</td>
<td>65.59</td>
</tr>
</tbody>
</table>

4.2. Econometric models and results for digital cameras

We first discuss our econometric model and empirical results for point-and-shoot digital cameras. We use the following econometric model:

\[
\ln(\text{Price}_i) = \alpha + \beta_1 \text{Rating}_i + \beta_2 \text{Rating}_i \times \text{Quality}_i + \beta_3 \text{OppositeQualityRating}_i \\
+ \beta_4 \text{OppositeQualityRating}_i \times \text{Quality}_i + \beta_5 \text{MP}_i + \beta_6 \text{OpticalZoom}_i + \beta_7 \ln(\text{DaysAvailable}_i) \\
+ \beta_8 \text{AmazonasSeller}_i + \varepsilon_i,  
\]  

(1)

where Rating\(_i\) is the average consumer star rating for Product \(i\); Quality\(_i\) is a dummy variable with a value of 1 if Product \(i\) is of high quality, and 0 otherwise; OppositeQualityRating\(_i\) is the average rating of all
products of the opposite quality in the same group; MP, is the maximum image resolution in mega pixels; OpticalZoom, is the optical zoom of the camera; ln(DaysAvailable,) is the natural logarithm of the number of days for which the product had been available on Amazon.com, since digital cameras generally exhibit a declining price trend over time; AmazonasSeller, is a dummy variable with a value of 1 if Amazon.com is the featured seller, and 0 otherwise.

We summarize the empirical results for digital cameras in Table 2. Overall, our model had a good fit with the data, as indicated by the R-square. Many of our control variables are significant and most signs are in the expected directions. One exception is ln(DaysAvailable), whose impact on ln(Price) is positive. This suggests that products that were available on Amazon.com longer were sold at higher prices—contradictory to the declining price trend that we usually observe for digital cameras. A close examination of our data revealed that many sellers were still selling older models at high prices, even though prices for digital cameras were declining as the technology became mature and newer models were available at lower prices.

Table 2. Empirical Results for Digital Cameras.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>3.493</td>
</tr>
<tr>
<td>Rating (β₁)</td>
<td>0.101</td>
</tr>
<tr>
<td>Rating·Quality (β₂)</td>
<td>0.075</td>
</tr>
<tr>
<td>OppositeQualityRating (β₃)</td>
<td>0.005</td>
</tr>
<tr>
<td>OppositeQualityRating·Quality (β₄)</td>
<td>-0.032</td>
</tr>
<tr>
<td>MP (β₅)</td>
<td>0.111</td>
</tr>
<tr>
<td>OpticalZoom (β₆)</td>
<td>0.074</td>
</tr>
<tr>
<td>ln(DaysAvailable) (β₇)</td>
<td>0.078</td>
</tr>
<tr>
<td>AmazonasSeller (β₈)</td>
<td>-0.236</td>
</tr>
<tr>
<td>N</td>
<td>355</td>
</tr>
<tr>
<td>R-Square</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Notes: ***, p<.01; **, p<.05, *, p<.10.
We next report the testing of our propositions. To facilitate comparisons, we summarize the signs of the expected effects in Table 3 and report our observed effects in Table 4. We calculated the significance of the sums of the coefficients based on their variances and the covariances. Our empirical results show that the signs of the effects of rating changes are all in the expected directions. Propositions 3(1) and 4(1) are supported or partially supported. The non-supported propositions are all due to insignificant results.

Table 3. Expected Signs of the Effects of Rating Changes Based on Our Propositions.

<table>
<thead>
<tr>
<th>Change in high quality product’s price</th>
<th>When High Quality Product(s)’s Rating Increase(s)</th>
<th>When Low Quality Product(s)’s Rating Increase(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in high quality product’s price</td>
<td>P3(1): +</td>
<td>P4(1): -</td>
</tr>
<tr>
<td>Change in low quality product’s price</td>
<td>P3(1): +</td>
<td>P4(1): +/-</td>
</tr>
</tbody>
</table>

Table 4. Proposition Testing Results for Digital Cameras.

<table>
<thead>
<tr>
<th>Change in high quality product’s price</th>
<th>When High Quality Product(s)’s Rating Increase(s)</th>
<th>When Low Quality Product(s)’s Rating Increase(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in high quality product’s price</td>
<td>$\beta_1 + \beta_2 = 0.101 + 0.075 = 0.176^{**}$ (P3(1) supported)</td>
<td>$\beta_3 + \beta_4 = 0.005 + (-0.032) = -0.027^{ns}$</td>
</tr>
<tr>
<td>Change in low quality product’s price</td>
<td>$\beta_3 = 0.005^{ns}$</td>
<td>$\beta_1 = 0.101^{*}$ (P4(1) partially supported)</td>
</tr>
</tbody>
</table>

Note: ns: not significant at the .10 level.

4.3. Econometric models and results for multivitamins

We next discuss our econometric model and empirical results for multivitamins. We use the following econometric model:

$$\ln(Price_i) = \alpha + \beta_1 \text{Rating}_i + \beta_2 \text{Rating}_i \cdot \text{Quality}_i + \beta_3 \text{OppositeQualityRating}_i$$

$$+ \beta_4 \text{OppositeQualityRating}_i \cdot \text{Quality}_i + \beta_5 \text{Servings}_i + \beta_6 \text{AmazonasSeller}_i + \epsilon_i,$$  

The definitions of the variables are similar to those in Model (1). One new variable, Servings, measures the number of daily servings that a multivitamin product contains.
We summarize the empirical results for multivitamins in Table 5. Overall, our model has a good fit with the data, as indicated by the R-square. Most of our control variables are significant and their coefficient estimates have the expected signs.

**Table 5. Empirical Results for Multivitamins.**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.380</td>
</tr>
<tr>
<td>Rating ($\beta_1$)</td>
<td>0.070</td>
</tr>
<tr>
<td>Rating-Quality ($\beta_2$)</td>
<td>0.560</td>
</tr>
<tr>
<td>OppositeQualityRating ($\beta_3$)</td>
<td>0.563</td>
</tr>
<tr>
<td>OppositeQualityRating-Quality ($\beta_4$)</td>
<td>-0.529</td>
</tr>
<tr>
<td>Servings ($\beta_5$)</td>
<td>0.001</td>
</tr>
<tr>
<td>AmazonasSeller ($\beta_6$)</td>
<td>0.064</td>
</tr>
<tr>
<td>N</td>
<td>139</td>
</tr>
<tr>
<td>R-Square</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 6 reports the proposition testing results. Our results show that three out of four parts of the propositions are supported. The non-significant results are Proposition 4(1) related to the impacts of a rating increase of the low quality product on the prices of the high quality products. In particular, the sign of $\beta_3 + \beta_4$ is insignificant and is not in the expected direction. That is, we do not find that an increase in the low quality product’s rating will prompt the competitor to lower the price of the high quality product. One possible reason may be that compared with the sample of digital cameras, our multivitamins sample had a higher average consumer rating with a larger standard deviation, suggesting a more diverse distribution of average consumer ratings. When a low quality product’s rating increased, it may still have a large gap with the ratings of the high quality products; as a result, sellers of the high quality products might not perceive the low quality product as a big threat, reducing their incentive to lower their prices. Another plausible reason may be that sellers of the high quality multivitamins want to avoid triggering intense price competition and hence refrain from cutting prices in response to the increase in the competitor’s product rating.
Table 6. Proposition Testing Results for Multivitamins.

<table>
<thead>
<tr>
<th></th>
<th>When High Quality Product(s)’s Rating Increase(s)</th>
<th>When Low Quality Product(s)’s Rating Increase(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Change in high quality product’s price</strong></td>
<td>$\beta_1+\beta_2=.070+.560=.630^{**}$ (P3(1) supported)</td>
<td>$\beta_3+\beta_4=.563+(-.529)=0.034^{ns}$</td>
</tr>
<tr>
<td><strong>Change in low quality product’s price</strong></td>
<td>$\beta_3=.563^{**}$ (P3(1) supported)</td>
<td>$\beta_1=.070^{ns}$ (P4(1) supported)</td>
</tr>
</tbody>
</table>

Overall, our empirical evidence provides strong support for our key propositions. The signs of all but one effect were in the expected directions, validating our analytical model.

5. Conclusions and Future Research

In this study, we have developed an analytical model to examine how consumer product ratings and reviews affect prices, sales, profits, and consumer surplus in both monopoly and duopoly markets with heterogeneous consumers. Using the consumer rating and price data collected from Amazon.com for point-and-shoot digital cameras and multivitamins, we provide empirical validation of some of our analytical results. Our research contributes to the extant literature in several ways. First, our research points out conditions under which consumer rating improvements will either benefit or hurt firms’ sales and profits, thus allowing us to better understand the contradictory findings in the extant empirical research literature regarding the impacts of consumer ratings on sales. Second, to the best of our knowledge, this paper is the first to analytically model both consumer reviews and ratings in a vertically rather than horizontally differentiated market with heterogeneous consumers and competition. Third, we take into consideration not only the consumer ratings and the informativeness of the reviews but also product quantifiability in examining the economic impacts of consumer ratings and reviews.

Our research results offer the following managerial insights on firm strategies. First, if a firm has monopoly power in a market, it should always take advantage of consumers’ higher WTPs due to higher consumer ratings and more informative reviews by raising its price to achieve higher profits. As an alternative to corporate marketing communications, consumer ratings and reviews allow consumers to
inform each other on the experience/credence attributes (i.e., the difficult to communicate/specify product features). Combined with product specifications on the search attributes, consumers can better infer the overall product qualities.

Second, in a vertically differentiated market, a firm’s optimal strategic response to consumer rating improvements depends on its relative product quality. When the firm’s product quality is relatively high, an improvement in its product rating will result in further separation between the perceived qualities of its product and those of its competitors’. This is equivalent to a more vertically differentiated market and will thus ease the competitive pressure. As a result, all firms in the market can strategically increase their prices and enjoy higher profits, even though their sales volumes may decline. In contrast, when the firm’s product is of a relatively low quality, an increase in its product rating will reduce the distance between the perceived qualities of its product and those of its competitors’, resulting in intensified competition. Depending on the ratio of the firm’s product quality to that of the high quality competitor, the firm’s optimal response to its rating’s improvement may be either to increase or to decrease its prices, and it will generate either higher or lower profits. The seller of the high quality product is always hurt by the rating improvement of its low quality competitor and is forced to reduce its prices making lower profits even though it may sell more units. In addition, our analysis shows that although consumers will always benefit from a rating improvement of the low quality product, they may either benefit or be hurt if the high quality product’s rating improves.

Third, we find that although the flexibility in adjusting prices may lead to higher profitability for both firms when consumer ratings improve for the high quality products, such flexibility will lead to the prisoner’s dilemma under other situations and will result in a price war with decreased profits for both firms. In the latter case, firms are better off when they cannot freely adjust their prices to respond to consumer rating changes.

Our study has limitations. In our model, consumer reviews and ratings are exogenous in that firms do not strategically influence or manipulate their reviews and ratings even though they will respond to rating changes. We have assumed that consumers do not make multiple purchases. Future research may
relax these assumptions and examine the dynamic effects of reviews and ratings. One may also study a more general setting with both horizontal and vertical product differentiation. In addition, we tested our propositions using data from only two product categories collected at one point in time. Future research can use panel data to examine if our propositions hold true for other products over an extended period of time.

Appendix

Proof of Proposition 1:

The optimal price, sales volume, and profit are given by

\[ P^* = \frac{c + Q(r, \eta, \lambda)}{2}, \quad q^* = \frac{Q(r, \eta, \lambda) - c}{2Q(r, \eta, \lambda)}, \quad \Pi^* = \frac{(Q(r, \eta, \lambda) - c)^2}{2Q(r, \eta, \lambda)}. \]

Differentiation with respect to \( r \) yields

\[ \frac{\partial P^*}{\partial r} = \frac{\partial Q^*}{\partial Q} = \frac{1 - \lambda}{2}, \quad \frac{\partial \Pi^*}{\partial Q} = \frac{1 - \lambda}{4Q(r, \eta, \lambda)} > 0, \]

and

\[ \frac{\partial q^*}{\partial r} = \frac{(1 - \lambda)c}{2Q(r, \eta, \lambda)} > 0, \quad \frac{\partial \Pi^*}{\partial Q} = \frac{1 - \lambda}{4Q^2(r, \eta, \lambda)} > 0. \]

\[ \square \]

Proof of Proposition 2:

\[ \frac{\partial^2 P^*}{\partial \eta \partial r} = \frac{\partial}{\partial \eta} \left( \frac{1 - \lambda}{2} \right) = \frac{1 - \lambda}{2} > 0, \]

\[ \frac{\partial^2 P^*}{\partial \lambda \partial r} = \frac{\partial}{\partial \lambda} \left( \frac{1 - \lambda}{2} \right) = \frac{-f_r(r, \eta)}{2} < 0. \]

\[ \square \]

Proof of Lemma 1:

Consumer purchasing decisions observe the individual rationality and incentive compatibility constraints. That is, a consumer of type \( v \) will purchase product \( i \) if

\[ u_i(v, P_i) = Q_i \cdot v - P_i \geq 0 \quad \text{and} \quad u_i(v, P_i) > u_j(v, P_j) \quad \text{for} \quad j \neq i. \]

From these constraints, it is straightforward to show that the consumer will buy product 1 if

\[ \frac{P_1 - P_2}{Q_1 - Q_2} \leq v \leq 1, \quad \text{and product 2 if} \quad \frac{P_2}{Q_2} \leq v < \frac{P_1 - P_2}{Q_1 - Q_2}. \]

Since consumers are assumed
to be uniformly distributed on \([0, 1]\), firms’ profit functions are given by 
\[ \Pi_1 = P_1 \left( 1 - \frac{P_1 - P_2}{Q_1 - Q_2} \right) \] and 
\[ \Pi_2 = P_2 \left( \frac{P_1 - P_2}{Q_1 - Q_2} - \frac{P_2}{Q_2} \right), \] respectively.

Each firm will select its own optimal price given the other’s price. The first order conditions are
\[ \frac{\partial \Pi_1}{\partial P_1} = \left( 1 - \frac{P_1 - P_2}{Q_1 - Q_2} \right) + \frac{P_1 - P_2}{Q_1 - Q_2} = 0 \] and 
\[ \frac{\partial \Pi_2}{\partial P_2} = \left( \frac{P_1 - P_2}{Q_1 - Q_2} - \frac{P_2}{Q_2} \right) - \frac{P_2}{Q_1 - Q_2} - \frac{P_2}{Q_2} = 0. \]

Solving these conditions simultaneously, we obtain the equilibrium prices (noting that the second order conditions for profit maximization clear hold).
\[ P_1^* = \frac{2(Q_1 - Q_2) \cdot Q_1}{4Q_1 - Q_2} \quad \text{and} \quad P_2^* = \frac{(Q_1 - Q_2) \cdot Q_2}{4Q_1 - Q_2}. \]

The equilibrium sales volumes (i.e., unit sales) and profits are then easily computed.
\[ q_1^* = \frac{2Q_1}{4Q_1 - Q_2}, \quad q_2^* = \frac{Q_1}{4Q_1 - Q_2}, \quad \Pi_1^* = \frac{4(Q_1 - Q_2) \cdot Q_1^2}{(4Q_1 - Q_2)^2}, \quad \Pi_2^* = \frac{(Q_1 - Q_2) \cdot Q_1 Q_2^2}{(4Q_1 - Q_2)^2}. \]

### Proof of Proposition 3:

By assumption, firm 1 has a higher perceived quality, i.e., \( Q_1(r_1, \eta_1, \lambda_1) > Q_2(r_2, \eta_2, \lambda_2) \). Recall that 
\[ Q_1(r_1, \eta_1, \lambda_1) = \lambda_1 + (1 - \lambda_1) f(r_1, \eta_1) \quad \text{and} \quad Q_2(r_2, \eta_2, \lambda_2) = \lambda_2 + (1 - \lambda_2) g(r_2, \eta_2). \] From the equilibrium quantities found in Lemma 1, we compute the comparative statics with respect to product ratings.
\[ \frac{\partial P_1^*}{\partial r_1} = \frac{\partial P_1^*}{\partial Q_1} \frac{\partial Q_1}{\partial r_1} = \frac{2(1 - \lambda_1) f(r, \eta) \cdot (4Q_1^2 - 2Q_1Q_2 + Q_2^2)}{(4Q_1 - Q_2)^2} \]

Since \( \lambda_1 \in (0,1) \) and \( f(r, \eta) > 0 \), and \( 4Q_1^2 - 2Q_1Q_2 + Q_2^2 = 3Q_1^2 + (Q_1 - Q_2)^2 > 0 \), we conclude that
\[ \frac{\partial P_1^*}{\partial r_1} > 0. \] Other comparative statics involve similar computations and arguments.
\[
\frac{\partial q_1^*}{\partial r_1} = \frac{\partial q_1^*}{\partial Q_1} \frac{\partial Q_1}{\partial r_1} = -\frac{2(1 - \lambda_1) f_r(r, \eta) \cdot Q_2}{(4Q_1 - Q_2)^2} < 0,
\]

\[
\frac{\partial \Pi_1^*}{\partial r_1} = \frac{\partial \Pi_1^*}{\partial Q_1} \frac{\partial Q_1}{\partial r_1} = \frac{4(1 - \lambda_1) f_r(r, \eta) \cdot Q_1 \cdot (4Q_1^2 - 3Q_1Q_2 + 2Q_2^2)}{(4Q_1 - Q_2)^3} > 0,
\]

\[
\frac{\partial P_2^*}{\partial r_1} = \frac{\partial P_2^*}{\partial Q_1} \frac{\partial Q_1}{\partial r_1} = \frac{3(1 - \lambda_1) f_r(r, \eta) \cdot Q_2^2}{(4Q_1 - Q_2)^2} > 0,
\]

\[
\frac{\partial q_2^*}{\partial r_1} = \frac{\partial q_2^*}{\partial Q_1} \frac{\partial Q_1}{\partial r_1} = -\frac{(1 - \lambda_1) f_r(r, \eta) \cdot Q_2}{(4Q_1 - Q_2)^2} < 0,
\]

\[
\frac{\partial \Pi_2^*}{\partial r_1} = \frac{\partial \Pi_2^*}{\partial Q_1} \frac{\partial Q_1}{\partial r_1} = \frac{(1 - \lambda_1) f_r(r, \eta) \cdot Q_2^2 \cdot (2Q_1 + Q_2)}{(4Q_1 - Q_2)^3} > 0.
\]

Proposition 3 follows immediately from the interpretations of these results. \(\square\)

**Proof of Proposition 4:**

Similar to the proof of Proposition 3, this proof requires computing the comparative statics with respect to the product rating \(r_2\).

\[
\frac{\partial P_1^*}{\partial r_2} = \frac{\partial P_1^*}{\partial Q_2} \frac{\partial Q_2}{\partial r_2} = \frac{-6(1 - \lambda_2) g_r(r, \eta) \cdot Q_1^2}{(4Q_1 - Q_2)^2}
\]

Since \(\lambda_i \in (0,1)\) and \(g_r(r, \eta) > 0\), we conclude that \(\frac{\partial P_1^*}{\partial r_2} < 0\). Other comparative statics involve similar computations and arguments.

\[
\frac{\partial q_1^*}{\partial r_2} = \frac{\partial q_1^*}{\partial Q_2} \frac{\partial Q_2}{\partial r_2} = \frac{2(1 - \lambda_2) \cdot g_r(r, \eta) \cdot Q_1}{(4Q_1 - Q_2)^2} > 0,
\]

\[
\frac{\partial \Pi_1^*}{\partial r_2} = \frac{\partial \Pi_1^*}{\partial Q_2} \frac{\partial Q_2}{\partial r_2} = \frac{-4(1 - \lambda_2) g_r(r, \eta) \cdot Q_1^2 \cdot (2Q_1 + Q_2)}{(4Q_1 - Q_2)^3} < 0,
\]

\[
\frac{\partial P_2^*}{\partial r_2} = \frac{\partial P_2^*}{\partial Q_2} \frac{\partial Q_2}{\partial r_2} = \frac{(1 - \lambda_2) g_r(r, \eta) \cdot (4Q_1^2 - 8Q_1Q_2 + 2Q_2^2)}{(4Q_1 - Q_2)^2} = \begin{cases} > 0, & \text{if } Q_2 < (4 - 2\sqrt{3})Q_1, \\ < 0, & \text{if } Q_2 > (4 - 2\sqrt{3})Q_1 \end{cases}
\]
\[ \frac{\partial q_2^*}{\partial r_2} = \frac{\partial q_2^*}{\partial Q_2} \frac{\partial Q_2}{\partial r_2} = \frac{(1-\lambda_2) \cdot g_r(r, \eta) \cdot Q_1}{(4Q_1 - Q_2)^2} > 0, \]

\[ \frac{\partial \Pi_2^*}{\partial r_2} = \frac{\partial \Pi_2^*}{\partial Q_2} \frac{\partial Q_2}{\partial r_2} = \frac{(1-\lambda_2) g_r(r, \eta) \cdot Q_1^2 \cdot (4Q_1 - 7Q_2)}{(4Q_1 - Q_2)^3} = \begin{cases} > 0, & \text{if } Q_2 < \frac{4Q_1}{7}, \\ < 0, & \text{if } Q_2 > \frac{4Q_1}{7}. \end{cases} \]

Proposition 4 follows immediately from the interpretations of these results. □

**Proof of Proposition 5:**

Consumer surplus is computed by

\[ CS = \int_l^s \left( Q_1 \cdot v - P_1^* \right) dv + \int_s^t \left( Q_2 \cdot v - P_2^* \right) dv, \]

where the integration bounds are \( l = \frac{P_1^* - P_2^*}{Q_1 - Q_2} \) and \( s = \frac{P_2^*}{Q_2} \). Substitution of the equilibrium prices and straightforward (but somewhat tedious) computations give us

\[ CS = \frac{Q_1^2 \cdot (4Q_1 + 5Q_2)}{2(4Q_1 - Q_2)^2}. \]

Comparative statics can be easily obtained:

\[ \frac{\partial CS}{\partial r_1} = \frac{\partial CS}{\partial Q_1} \frac{\partial Q_1}{\partial r_1} = \frac{(1-\lambda_1)f_r(r, \eta) \cdot (8Q_1^2 - 6Q_1Q_2 - 5Q_2^2)}{2(4Q_1 - Q_2)^3} = \begin{cases} > 0, & \text{if } Q_2 < \frac{4Q_1}{5}, \\ < 0, & \text{if } Q_2 > \frac{4Q_1}{5}. \end{cases} \]

\[ \frac{\partial CS}{\partial r_2} = \frac{\partial CS}{\partial Q_2} \frac{\partial Q_2}{\partial r_2} = \frac{(1-\lambda_2) g_r(r, \eta) \cdot Q_1^2 \cdot (28Q_1 + 5Q_2)}{2(4Q_1 - Q_2)^3} > 0. \]

Proposition 5 follows from the interpretations of these comparative statics. □

**Proof of Proposition 6:**

Suppose that firms cannot adjust their prices when firm 2’s product rating changes. Then, firms’ profit functions are

\[ \Pi_1 = P_1^* \left( 1 - \frac{P_1^* - P_2^*}{Q_1 - Q_2} \right) \quad \text{and} \quad \Pi_2 = P_2^* \left( \frac{P_1^* - P_2^* - P_2^*}{Q_1 - Q_2} \right), \]

where \( P_1^* \) are fixed. We take
the partial derivatives of these profit functions with respect to $r_2$ keeping $P^*_i$ fixed, and then substitute

$P^*_i$ expressions computed in Lemma 1. The results are simplified to

$$\frac{\partial \Pi_1}{\partial r_2} \bigg|_{P^*_i} = \frac{\partial Q_2}{\partial r_2} \cdot \frac{\partial \Pi_1}{\partial Q_2} \bigg|_{P^*_i} = \frac{(1 - \lambda_2)g_r(r, \eta) \cdot P^*_i \cdot (P^*_i - P^*_2)}{(Q_1 - Q_2)^2} = \frac{2(1 - \lambda_2)g_r(r, \eta) \cdot Q_1 \cdot (2Q_1 - Q_2)}{(4Q_1 - Q_2)^2},$$

$$\frac{\partial \Pi_2}{\partial r_2} \bigg|_{P^*_i} = \frac{\partial Q_2}{\partial r_2} \cdot \frac{\partial \Pi_2}{\partial Q_2} \bigg|_{P^*_i} = (1 - \lambda_2)g_r(r, \eta) \cdot P^*_2 \cdot \left( \frac{P^*_i - P^*_2}{(Q_1 - Q_2)^2} + \frac{P^*_2}{Q_2} \right) = \frac{(1 - \lambda_2)g_r(r, \eta) \cdot Q_1^2}{(4Q_1 - Q_2)^2} > 0.$$

Recall that in the proof of Proposition 4, we found that if firms can vary their prices flexibly,

$$\frac{\partial \Pi_2^*}{\partial r_2} \bigg|_{P^*_i} = \frac{\partial Q_2}{\partial r_2} \cdot \frac{\partial \Pi_2}{\partial Q_2} \bigg|_{P^*_i} = \frac{(1 - \lambda_2)g_r(r, \eta) \cdot Q_1^2 \cdot (4Q_1 - 7Q_2)}{(4Q_1 - Q_2)^3} = \begin{cases} > 0, & \text{if } Q_2 < \frac{4Q_1}{7}, \\ < 0, & \text{if } Q_2 > \frac{4Q_1}{7}. \end{cases}$$

If $Q_2 > \frac{4Q_1}{7}$, firm 2 will see its profit rise if neither firm can adjust their prices but drop if both can change their prices. If $Q_2 < \frac{4Q_1}{7}$, we easily show $\frac{\partial \Pi_2}{\partial r_2} \bigg|_{P^*_i} > \frac{\partial \Pi_2^*}{\partial r_2}$ because

$$\frac{\partial \Pi_2}{\partial r_2} \bigg|_{P^*_i} = \frac{(1 - \lambda_2)g_r(r, \eta) \cdot Q_1^2}{(4Q_1 - Q_2)^2} = \frac{4Q_1 - Q_2}{4Q_1 - 7Q_2} > 1.$$  Thus, when its rating improves, firm 2 is better off if prices are inflexible.

$$\frac{\partial \Pi_1}{\partial r_2} \bigg|_{P^*_i} = -\frac{2(1 - \lambda_2)g_r(r, \eta) \cdot Q_1 \cdot (2Q_1 - Q_2)}{(4Q_1 - Q_2)^2} = \frac{(4Q_1 - Q_2)(2Q_1 - Q_2)}{2Q_1 \cdot (2Q_1 + Q_2)} < 1,$$  if $Q_1 < \frac{(2 + \sqrt{3})Q_2}{2}$. Thus, if $Q_1$ is not too high, firm 1’s profit decrease due to firm 2’s rating improvement will be lower if prices are inflexible than if prices can freely adjust.

\[\square\]
References


Chen, P.-Y., S. Dhanasobhon, M. Smith, D. 2006. All reviews are not created equal.


